時間序列模型在農業的運用-以甘藍為例

農委會統計室

吳金擇

2022 年 3 月 4 日

問題起源

- 甘藍價格變動
- 價格需求彈性

$$E^D = -\frac{dQ^D}{dP} \times \frac{P}{Q^D} = -\frac{d\ln(Q^D)}{d\ln(P)}$$

$$\frac{d(TR)}{dP} = \frac{d(TE)}{dP} = Q^D(1 - E^D)$$

甘藍新聞 (1/2)

高麗菜價崩盤 批發價 1 公斤跌破 6 元

2018/04/17 12:51

繼洋蔥價跌之後，高麗菜也傳出慘跌，一顆高山高麗菜只要 20 多元，平地種的甚至一顆不用 10 元，讓民眾樂的搶便宜，但菜農可就笑不出來，因為高麗菜供過於求，批發價格每公斤不到 6 元，乾脆不請工人採收，開放民眾自己來摘菜。

1 顆 200 變 10 元 農民憂高麗菜價崩盤

2019/11/18 17:00

今年 8 月高麗菜一度喊出一顆 200 元的高價，不料經 2 個月，高麗菜價一路下跌，前天（17 日）台北農產運銷公司拍出 11 元的價格，產地甚至出現自採 1 顆 10 元情況，農民擔憂政府的預警系統失靈，去年崩盤狀況恐再上演。

甘藍新聞 (2/2)

高麗菜超種 2 度警示 價格卻較往年倍增

2020-11-16 19:58

民眾在市場精挑細選要買些菜，目光幾乎都放在葉菜類，架上的高麗菜卻是乏人問津，攤販透露，今年高麗菜價格一直居高不下，已經 11 月中旬，價格仍在 40 元上下，跟往年一斤 20 多元左右價格相差一倍，消費者大喊吃不消，買不下去。

1 顆高麗菜 200 多元！市場婆婆媽媽唉唉叫「錢變薄」

2021/09/04 12:51

傳統市場最常見的高麗菜，大顆秤斤買要價 225 元，挑小顆一點的也要 150 多元，等於是 2 個甚至 3 個便當價，婆婆媽媽唉唉叫，有人加減買，有人乾脆只買 2 把或 3 把 50 元的葉菜類青菜。

現況說明

- 業務單位需求 - 統計模型
- 點估計限制
- 民眾不理解信賴區間

文獻回顧

- 模型
  - 國內
  - 國外
- 扇形圖 (Fan Chart)
  - Bank of England
  - OECD
  - OTHERS

模型 (1/2)

- 國內
  - 王品翔 (2018), " 應用倒傳遞類神經網路於農產品價格預測並探討其產銷模式 - 以甘藍為例", 碩士論文, 國立勤益科技大學
  - 翟柏森 (2017), " 應用開放資料預測農產品菜價之研究：以甘藍為例", 碩士論文, 國立臺灣師範大學
  - 劉鋼 (2021), " 農產價格", 教學課程, 國立臺灣大學

模型 (2/2)

- 國外
  - Lorenzo Menculini et al.(2021), "Comparing Prophet and Deep Learning to ARIMA in Forecasting Wholesale Food Prices"
  - Sindhuja T et al.(2021), "Vegetable Price Prediction using ARIMA"
  - Zhiyuan Chen et al.(2021), "Automated Agriculture Commodity Price Prediction System with Machine Learning Techniques"
  - U.S. Department of Agriculture(USDA) PubAg

扇形圖 (1/3)

- Bank of England

Chart 1.1: GDP projection based on market interest rate expectations, other policy measures as announced

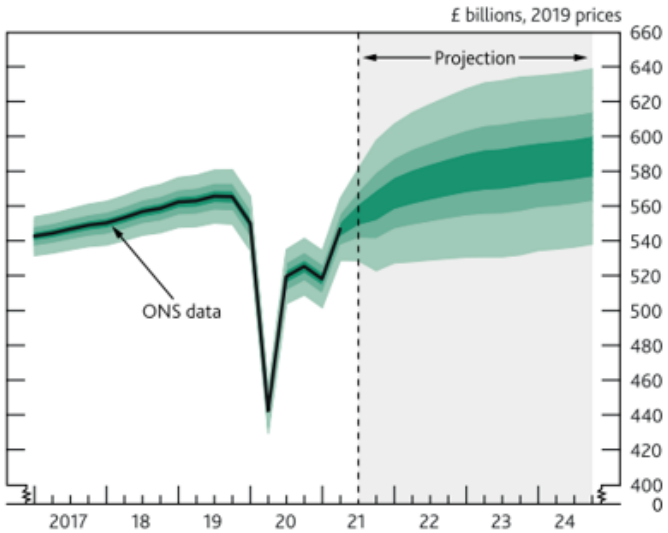
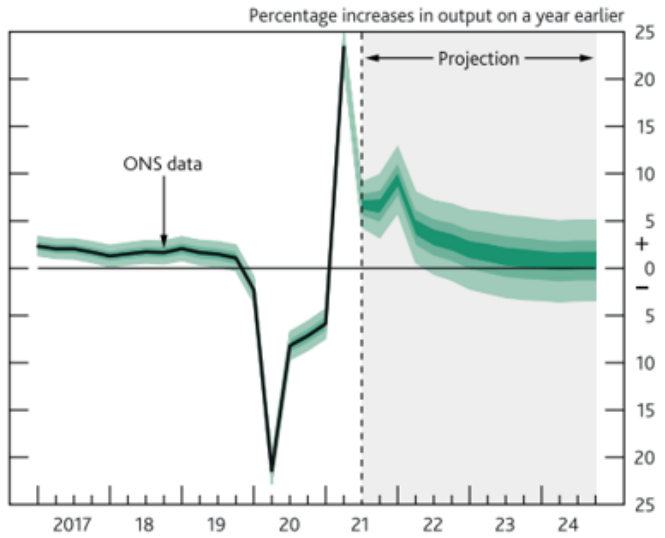


Chart 1.2: GDP growth projection based on market interest rate expectations, other policy measures as announced



扇形圖 (2/3)

- OECD - Economics Department Working Papers
  - Designing ***Fan Charts*** For GDP Growth Forecasts to Better Reflect Downturn Risks, 17 Nov 2017
  - ***Fan Charts*** Around GDP Projections Based on Probit Models of Downturn Risk, 11 Dec 2018
  - Calibrating GDP ***Fan Charts*** Using Probit Models with a Comparison to the Approaches of The Bank of England and Riksbank, 08 Mar 2019
  - Using Probit Models of Downturn Risk to Calibrate GDP ***Fan Charts*** for New Zealand, 08 Mar 2019

扇形圖 (3/3)

- OTHERS
  - Eurostat(2021) - Measuring and Communicating Uncertainty in Official Statistics: State of the Art and Perspectives
  - Journal of Official Statistics(2021) - Measuring and Communicating the Uncertainty in Official Economic Statistics
  - The Economist(2020) - The pandemic could lead statisticians to change how they estimate GDP
  - 中央銀行 (2021) - 連鎖法衡量下之台灣短中期通膨與經濟成長預測
  - United Nations Department of Economic and Social Affairs - World Population Prospects

資料來源

- 甘藍批發市場價格
- 甘藍育苗場供苗情形
- 氣象觀測資料

農產品批發市場交易行情站

敏感性蔬菜種植登記暨預警系統

交通部中央氣象局觀測資料

理論模型

- 模型
- 扇形圖

模型

- X13-ARIMA
- SARIMAX
- fbprophet
- (GARCH、XGBoost、LSTM、DeepAR、TFT ...)

X13-ARIMA(1/3)

三、我國 GDP 支出面季節調整

(一)沿革

1966 年 7 月第 18 次國民所得統計評審委員會(以下簡稱評審會)之評審委員建議籌劃辦理按季估計國民所得，亦同時決議應著手研究季節調整方法，開啟我國季節調整新頁。1971 年 5 月第 47 次評審會本總處首次將調整季節變動因素試算結果提請評審會核議。1981 年 6 月，採美國 X-11Q 季節調整軟體進行編製作業，公布 1961~1979 年各季各業生產及國民生產毛額處分之季節調整資料。隨後，每隔 5 年配合 5 年修正作業及基期年更換，於 1986 年、1991 年及 1996 年採加拿大 Dr. Dagum 發展之 X-11-ARIMA 進行季節調整作業，2001 年則改採美國 X-12-ARIMA 軟體，編製季節調整數列。考量每 5 年編布 1 次，景氣轉折變動皆已成歷史，且無法滿足使用者即時需求，加以衡酌國際趨勢，經 2009 年 8 月第 206 次評審會決議，自該月起按季發布國民所得支出面季節調整數列統計，供各界參考應用。目前我國所使用的季節調整軟體為美國 X-13A-S。

X13-ARIMA(2/3)

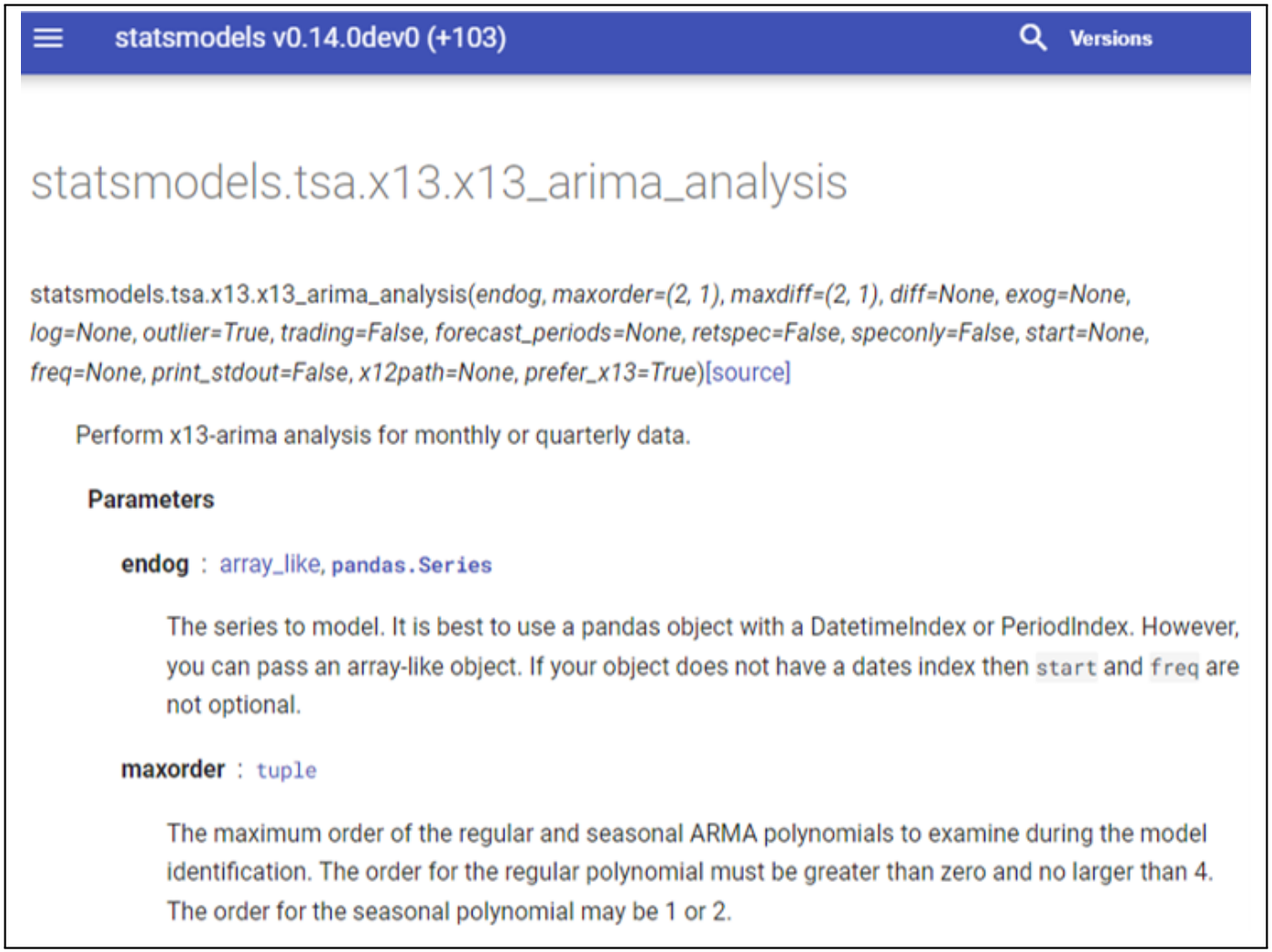
表29　歷年人力資源調查重要項目季節調整結果

單位：千人；%

年月別 Year & month	勞動力 Labor force			就業者 Employed		
	計 Both sexes	男 Male	女 Female	計 Both sexes	男 Male	女 Female
110年平均 Ave., 2021						
1 月 Jan.	11 967	6 633	5 334	11 518	6 377	5 141
2 月 Feb.	11 967	6 632	5 335	11 521	6 378	5 143
3 月 Mar.	11 975	6 632	5 343	11 529	6 380	5 149
4 月 Apr.	11 978	6 631	5 348	11 534	6 381	5 153
5 月 May	11 908	6 572	5 336	11 414	6 297	5 117
6 月 June	11 877	6 559	5 318	11 312	6 250	5 062
7 月 July	11 883	6 567	5 316	11 365	6 292	5 074
8 月 Aug.	11 869	6 564	5 304	11 385	6 298	5 086
9 月 Sept.	11 890	6 571	5 319	11 424	6 312	5 112
10 月 Oct.	11 901	6 579	5 322	11 444	6 326	5 118

註：本季節調整作業係採用X-13-ARIMA-SEATS季節調整軟體。

X13-ARIMA(3/3)



statsmodels v0.14.0dev0 (+103) Versions

statsmodels.tsa.x13.x13_arima_analysis

statsmodels.tsa.x13.x13_arima_analysis(*endog*, *maxorder=(2, 1)*, *maxdiff=(2, 1)*, *diff=None*, *exog=None*, *log=None*, *outlier=True*, *trading=False*, *forecast_periods=None*, *retspec=False*, *speconly=False*, *start=None*, *freq=None*, *print_stdout=False*, *x12path=None*, *prefer_x13=True*)[source]

Perform x13-arima analysis for monthly or quarterly data.

Parameters

endog : array_like, `pandas.Series`

The series to model. It is best to use a pandas object with a DatetimeIndex or PeriodIndex. However, you can pass an array-like object. If your object does not have a dates index then `start` and `freq` are not optional.

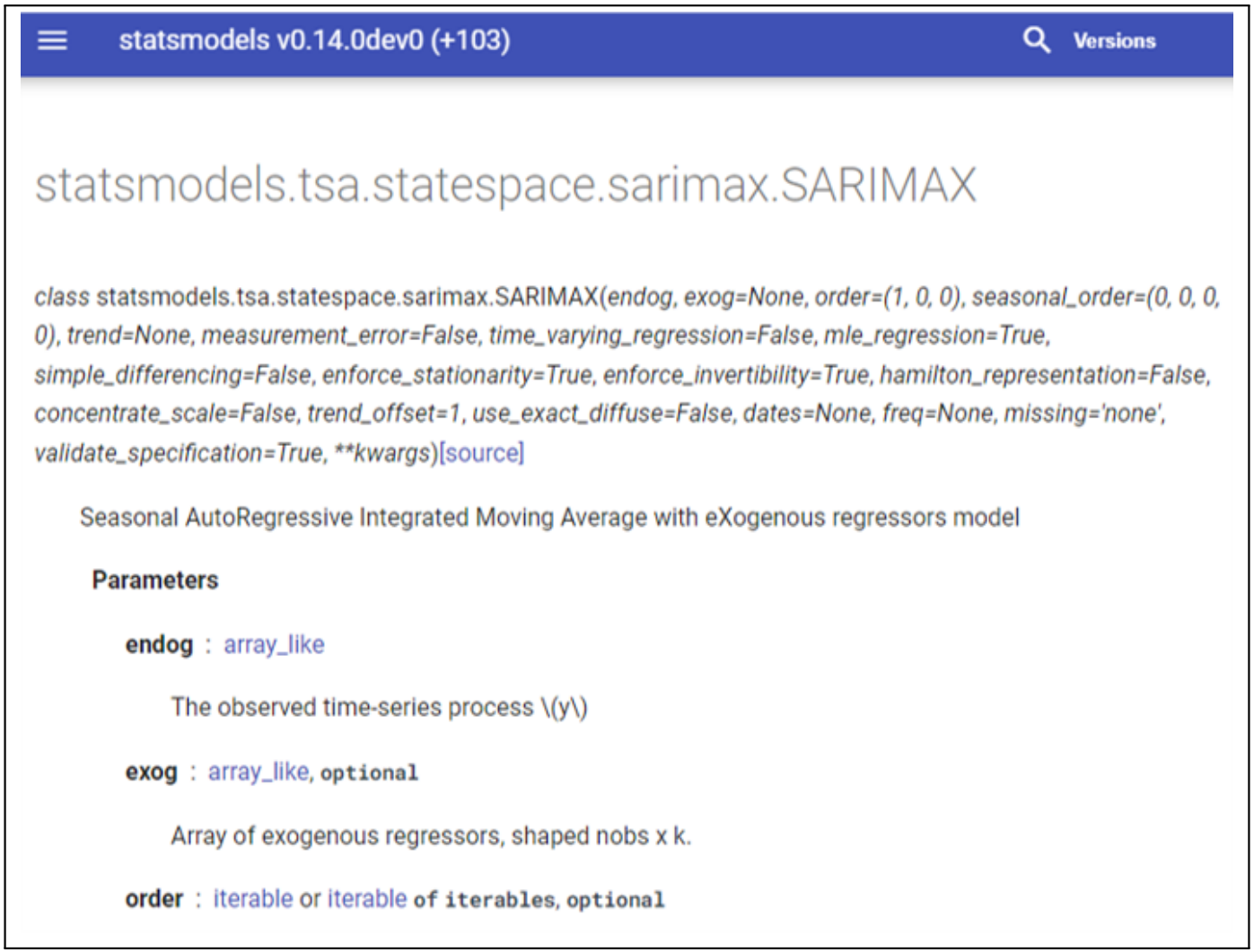
maxorder : `tuple`

The maximum order of the regular and seasonal ARMA polynomials to examine during the model identification. The order for the regular polynomial must be greater than zero and no larger than 4. The order for the seasonal polynomial may be 1 or 2.

SARIMAX(1/2)

- S - Seasonal
- AR - Auto Regression
- I - Integrated
- MA - Moving Average
- X - Exogenous

SARIMAX(2/2)



statsmodels v0.14.0dev0 (+103) Versions

statsmodels.tsa.statespace.sarimax.SARIMAX

class statsmodels.tsa.statespace.sarimax.SARIMAX(*endog*, *exog=None*, *order=(1, 0, 0)*, *seasonal_order=(0, 0, 0, 0)*, *trend=None*, *measurement_error=False*, *time_varying_regression=False*, *mle_regression=True*, *simple_differencing=False*, *enforce_stationarity=True*, *enforce_invertibility=True*, *hamilton_representation=False*, *concentrate_scale=False*, *trend_offset=1*, *use_exact_diffuse=False*, *dates=None*, *freq=None*, *missing='none'*, *validate_specification=True*, ***kwargs*)[source]

Seasonal AutoRegressive Integrated Moving Average with eXogenous regressors model

Parameters

endog : array_like

The observed time-series process \(y\)

exog : array_like, **optional**

Array of exogenous regressors, shaped nobs x k.

order : iterable or iterable **of iterables**, **optional**

fbprophet(1/2)

Forecasting at Scale

Sean J. Taylor*†
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and

Benjamin Letham†
Facebook, Menlo Park, California, United States
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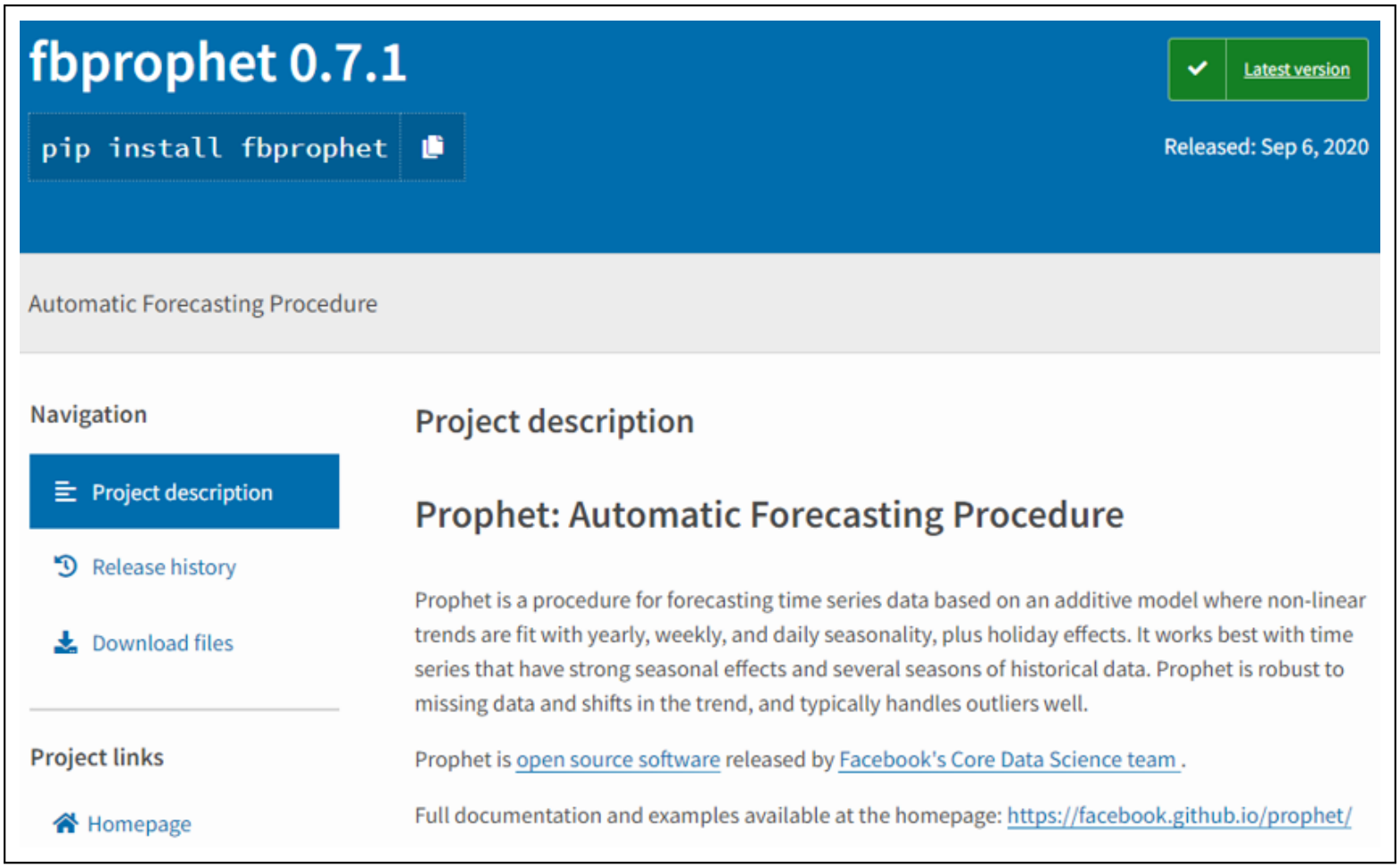
Abstract

Forecasting is a common data science task that helps organizations with capacity planning, goal setting, and anomaly detection. Despite its importance, there are serious challenges associated with producing reliable and high quality forecasts – especially when there are a variety of time series and analysts with expertise in time series modeling are relatively rare. To address these challenges, we describe a practical approach to forecasting "at scale" that combines configurable models with analyst-in-the-loop performance analysis. We propose a modular regression model with interpretable parameters that can be intuitively adjusted by analysts with domain knowledge about the time series. We describe performance analyses to compare and evaluate forecasting procedures, and automatically flag forecasts for manual review and adjustment. Tools that help analysts to use their expertise most effectively enable reliable, practical forecasting of business time series.

Keywords: Time Series, Statistical Practice, Nonlinear Regression

fbprophet(2/2)



實作

1. 安裝套件
2. 載入資料
3. 檢視資料
4. 切割資料
5. 資料分析
6. 模型
   - SARIMAX
   - prophet
7. 評估
8. 扇形圖

扇形圖與時間序列模型

吳金擇
農委會統計室

2022 年 3 月 4 日

1 安裝套件

[1]:
```
# pip install prophet
# pip install pmdarima
```

[2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.tsa.statespace.sarimax import SARIMAX
import pmdarima as pm
from prophet import Prophet
```

2 載入資料

[3]:
```
df_c = pd.read_excel('cabbage.xlsx', index_col='date', parse_dates=True)
df_m = pd.read_csv('mvrs.csv', index_col='date', parse_dates=True)
cols = ['ObsTime', 'Temperature', 'Precp']
df_w = pd.read_csv('weather.csv', usecols=cols, index_col='ObsTime', parse_dates=True)
df_w.index.name = 'date'
```

3 檢視資料

3.1 甘藍

[4]:
```
display(df_c.head())
df_c.price.plot()
plt.title('每日甘藍批發價格')
plt.ylabel('元/公斤')
plt.xlabel('')
plt.show()
```

date	price	quantity
2011-01-01	8.645042	297507
2011-01-02	8.257309	272084
2011-01-04	9.171932	235652
2011-01-05	9.839765	203173
2011-01-06	10.080478	194846

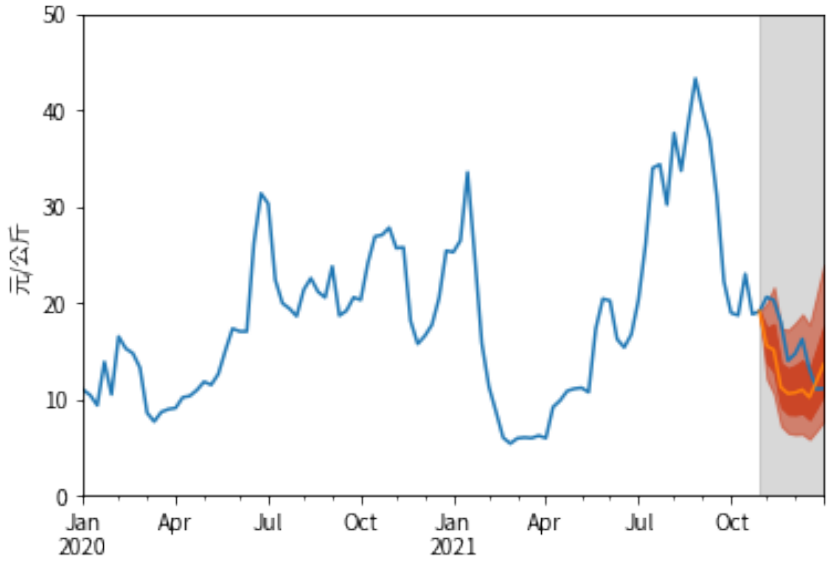
1

總結

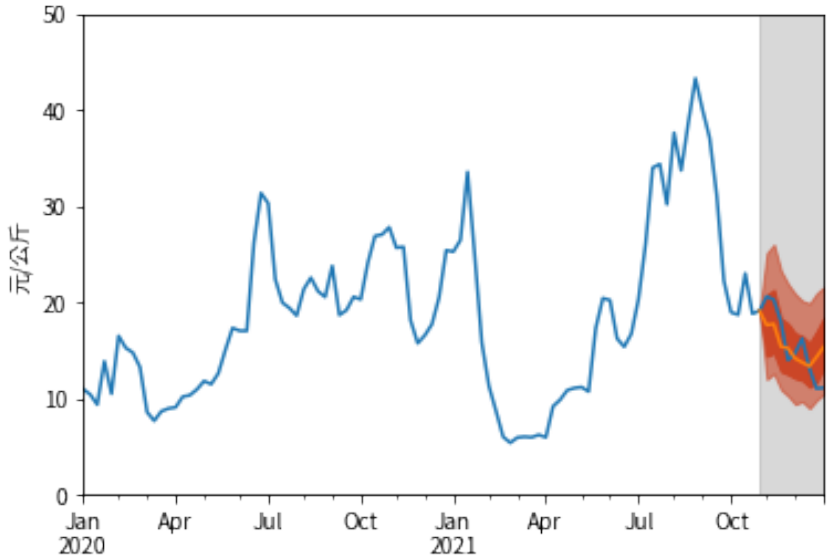
- 結論
- 研究限制
- 未來展望

結論 (1/2)

SARIMAX

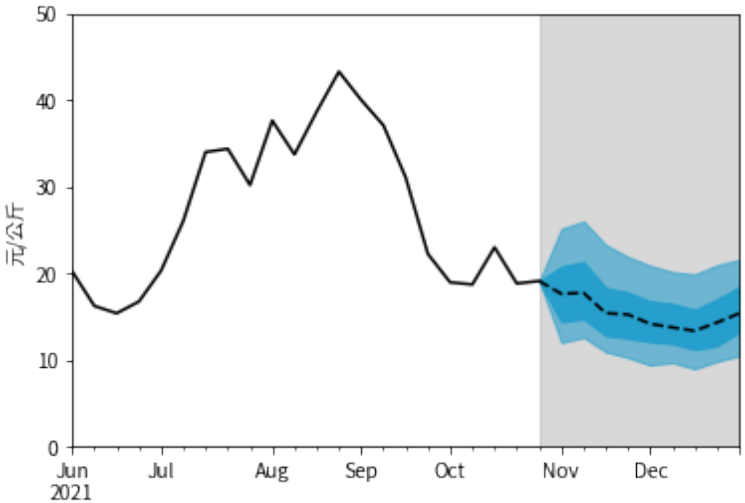


prophet



結論 (2/2)

- 扇形圖
  - Quantifying and visualizing future uncertainties.
  - Measuring and communicating uncertainties.



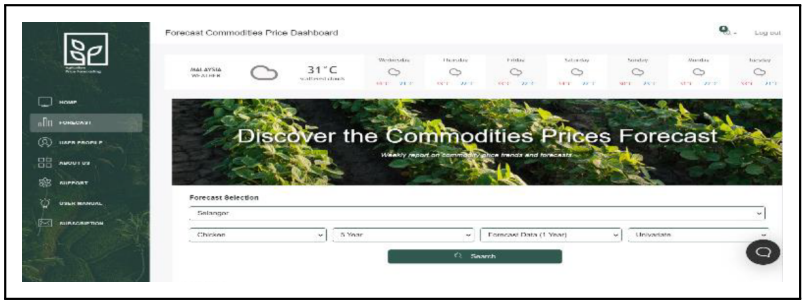
研究限制

- 變數與模型
  - 變數設定
  - 模型挑選
  - 參數調整

圖片來源: 沈昇勳同學

未來展望

- A Web-based Automated System



THANKS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推動
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總決算
年度
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疫情
人口
特種
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政策
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距離
系統
近年
運用
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辦公
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特別
指數
處理
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扇形圖與時間序列模型

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plt.xlabel('')
plt.show()
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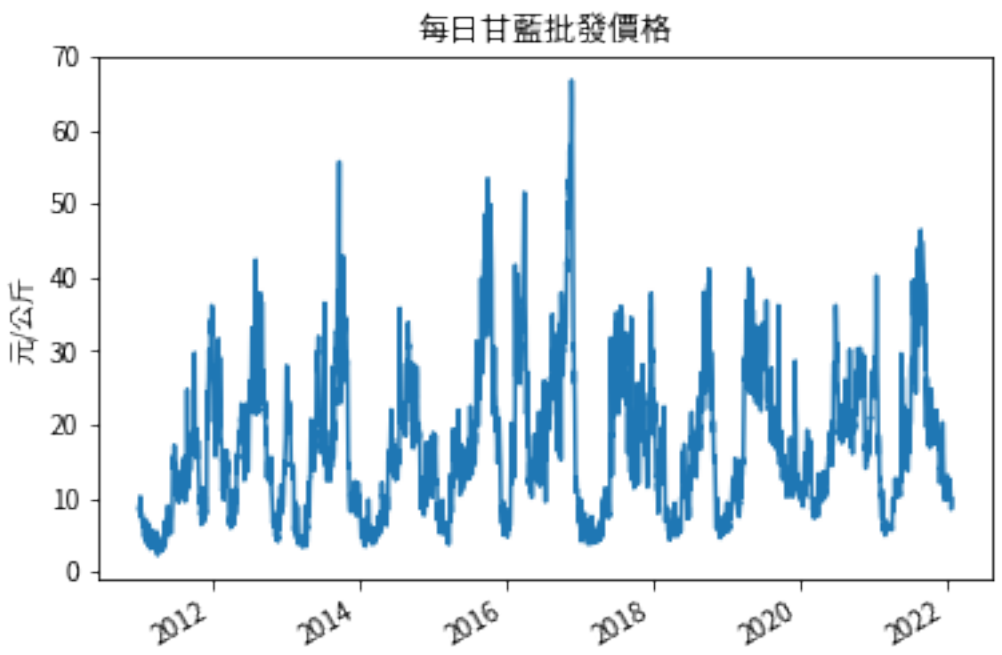
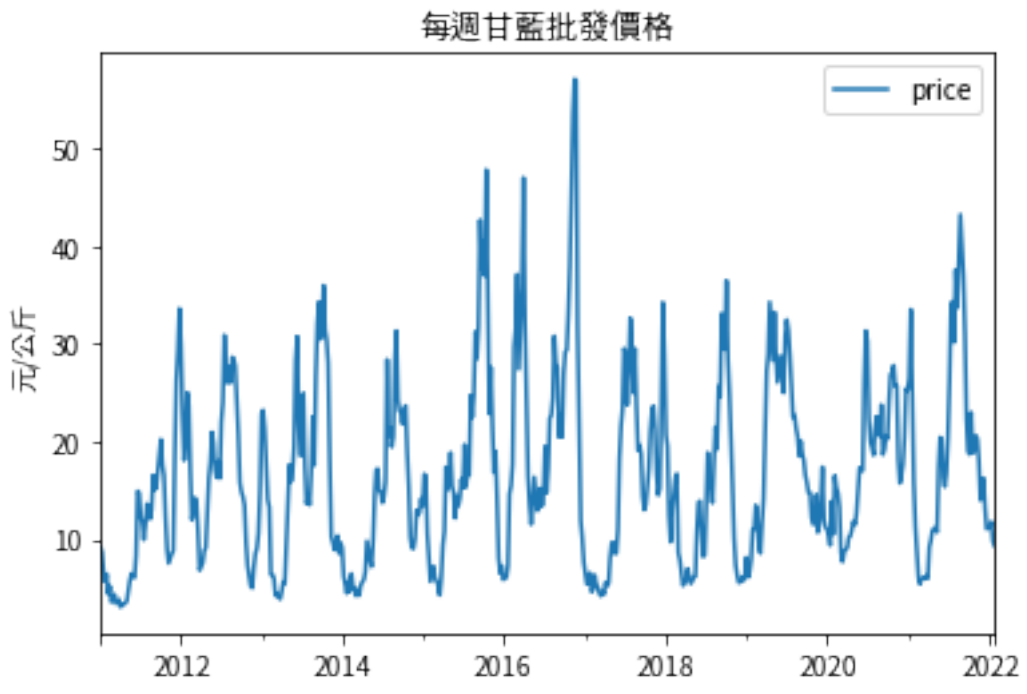


[5]:
```
df_c['pq'] = df_c.price * df_c.quantity
df_c = df_c.resample('W-Fri', label='right').sum()
df_c.price = df_c.pq / df_c.quantity
df_c = df_c.drop(columns=['quantity', 'pq'])
```

[6]:
```
display(df_c.head())
display(df_c.tail())
df_c.plot()
plt.title('每週甘藍批發價格')
plt.ylabel('元/公斤')
plt.xlabel('')
plt.show()
```

```
              price
date
2011-01-07  9.229535
2011-01-14  8.637490
2011-01-21  6.727316
2011-01-28  5.659670
2011-02-04  6.425445
```

```
               price
date
2021-12-31  11.118384
2022-01-07  11.310858
2022-01-14  11.804070
2022-01-21   9.820779
2022-01-28   9.356194
```



3.2 甘藍育苗

[8]:
```
df_m.head() # 千株
```

[8]:
```
            LA_num
date
2011-01-05  6020.64
2011-01-15  5809.97
2011-01-25  6209.84
2011-02-05  4715.74
2011-02-15  4997.61
```

[9]:
```
df_m = df_m.resample('W-Fri', label='right').nearest()
df_m.head()
```

[9]:
```
            LA_num
date
2011-01-07  6020.64
2011-01-14  5809.97
2011-01-21  6209.84
2011-01-28  6209.84
2011-02-04  4715.74
```

[10]:
```
df_m = df_m.shift(9)
display(df_m.head())
df_m.tail()
```

```
           LA_num
date
2011-01-07    NaN
2011-01-14    NaN
2011-01-21    NaN
2011-01-28    NaN
2011-02-04    NaN
```

[10]:
```
            LA_num
date
2021-12-03  6514.52
2021-12-10  6514.52
2021-12-17  6806.48
2021-12-24  7487.16
2021-12-31  7487.16
```

3.3 天氣

[11]:
```
df_w.head() # 攝氏、毫米
```

[11]:
```
            Temperature  Precp
date
2011-01-01         11.3    0.0
2011-01-02         14.1    0.0
2011-01-03         13.5    3.0
2011-01-04         13.1    0.1
2011-01-05         16.8    1.0
```

[12]:
```
df_w = df_w.resample('W-Fri', label='right').mean()
display(df_w.head())
df_w.tail()
```

```
            Temperature     Precp
date
2011-01-07    13.428571  0.714286
2011-01-14    13.442857  7.614286
2011-01-21    14.014286  1.000000
2011-01-28    15.071429  0.385714
2011-02-04    14.071429  0.985714
```

[12]:
```
            Temperature     Precp
date
2021-12-03    19.185714  1.857143
2021-12-10    19.514286  2.785714
2021-12-17    20.514286  0.785714
2021-12-24    18.300000  3.428571
2021-12-31    15.700000  0.714286
```

3.4 合併

[13]:
```
df = pd.concat([df_c, df_m, df_w], axis=1)
df = df['2011-03-11':'2021']
dfall = df.copy()
display(df.head())
df.tail()
```

```
              price   LA_num  Temperature     Precp
date
2011-03-11  4.351267  6020.64    15.585714  8.000000
2011-03-18  3.421378  5809.97    17.071429  0.542857
2011-03-25  3.442451  6209.84    17.428571  0.128571
2011-04-01  3.818407  6209.84    16.357143  8.342857
2011-04-08  3.057363  4715.74    19.542857  0.428571
```

[13]:
```
              price   LA_num  Temperature     Precp
date
2021-12-03  14.733079  6514.52    19.185714  1.857143
2021-12-10  16.248890  6514.52    19.514286  2.785714
2021-12-17  13.145006  6806.48    20.514286  0.785714
2021-12-24  11.053453  7487.16    18.300000  3.428571
2021-12-31  11.118384  7487.16    15.700000  0.714286
```

[14]:
```
df['ln_price'] = np.log(df.price)
```

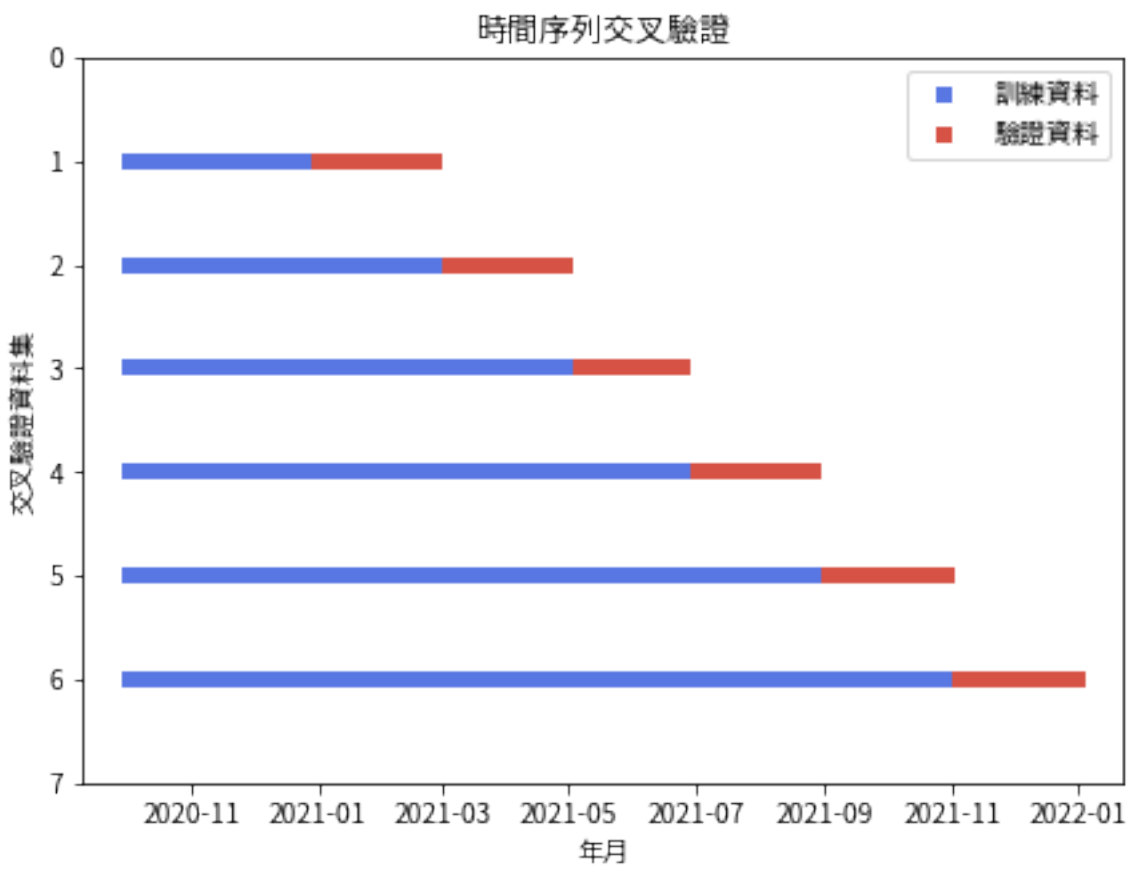
[15]:
```
print(df.isna().sum())
```

```
price          0
LA_num         0
Temperature    0
Precp          0
ln_price       0
dtype: int64
```

4 切割資料

[16]:
```
tr_splits = ['2020-12', '2021-02', '2021-04',
             '2021-06', '2021-08', '2021-10', '2021-12']
va_splits = ['2021-01', '2021-03', '2021-05',
             '2021-07', '2021-09', '2021-11']
trs, vas = [], []

for i in range(6):
    trs.append(df[:tr_splits[i]])
    vas.append(df[va_splits[i]:tr_splits[i+1]])
```

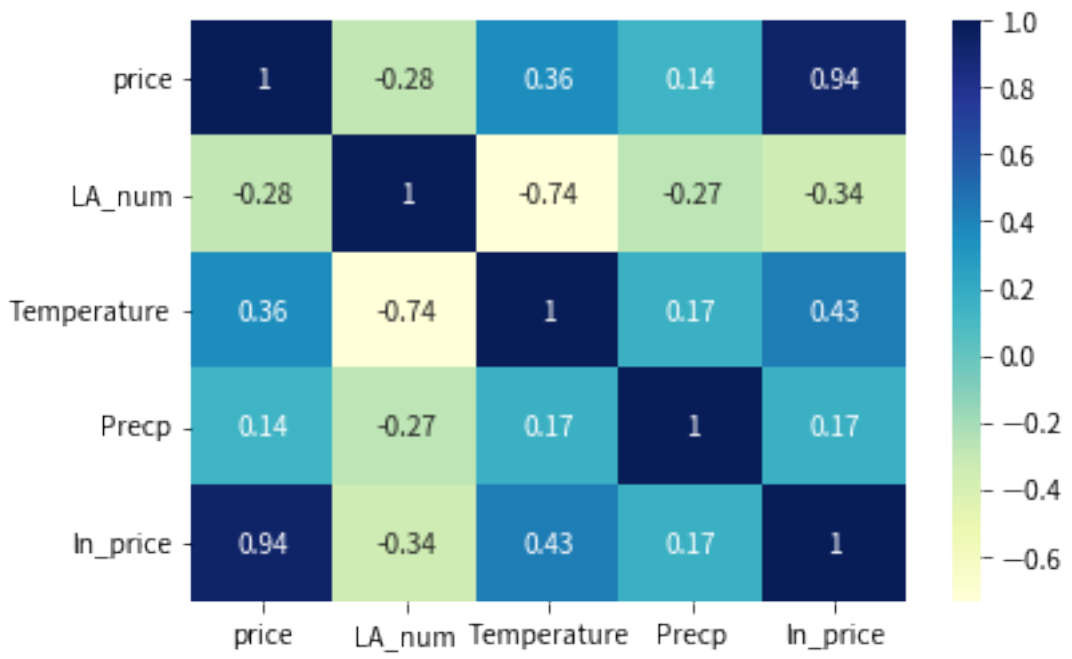
[17]:
```
fig, ax = plt.subplots(figsize=(7, 5))
for ii, (r, v) in enumerate(zip(trs, vas), 1):
    l1 = ax.scatter(r['2020-10':].index, [ii]*len(r['2020-10':]),
                    c=[plt.cm.coolwarm(.1)], marker='_', lw=6)
    l2 = ax.scatter(v.index, [ii]*len(v),
                    c=[plt.cm.coolwarm(.9)], marker='_', lw=6)
    ax.set(title='時間序列交叉驗證', ylim=[7, 0],
           xlabel='年月', ylabel='交叉驗證資料集',)
    ax.legend([l1, l2], ['訓練資料', '驗證資料'])
```



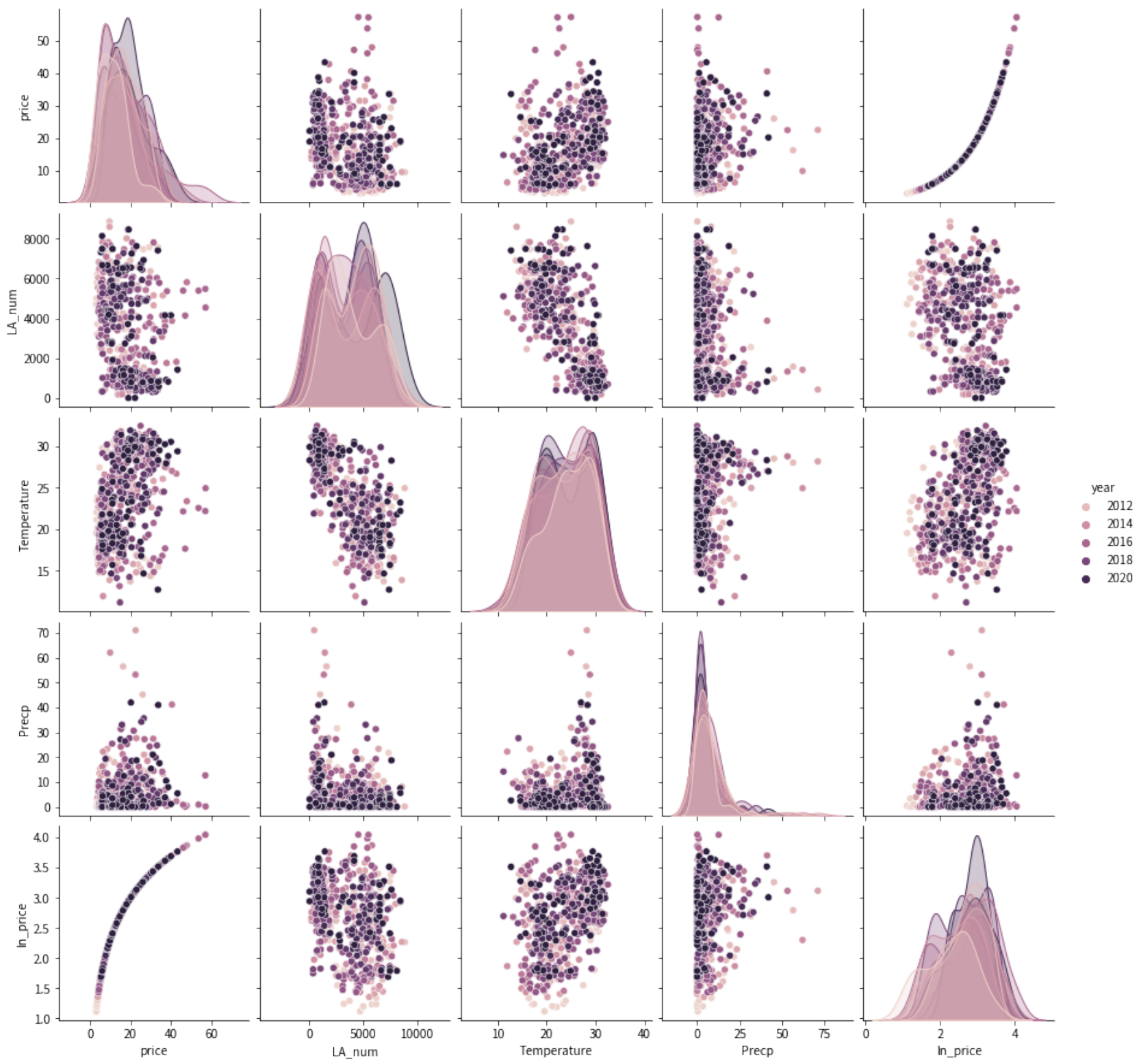
5 資料分析

5.1 Visualization

[18]:
```
import seaborn as sns
sns.heatmap(df.corr(), annot=True, cmap='YlGnBu')
plt.show()
```

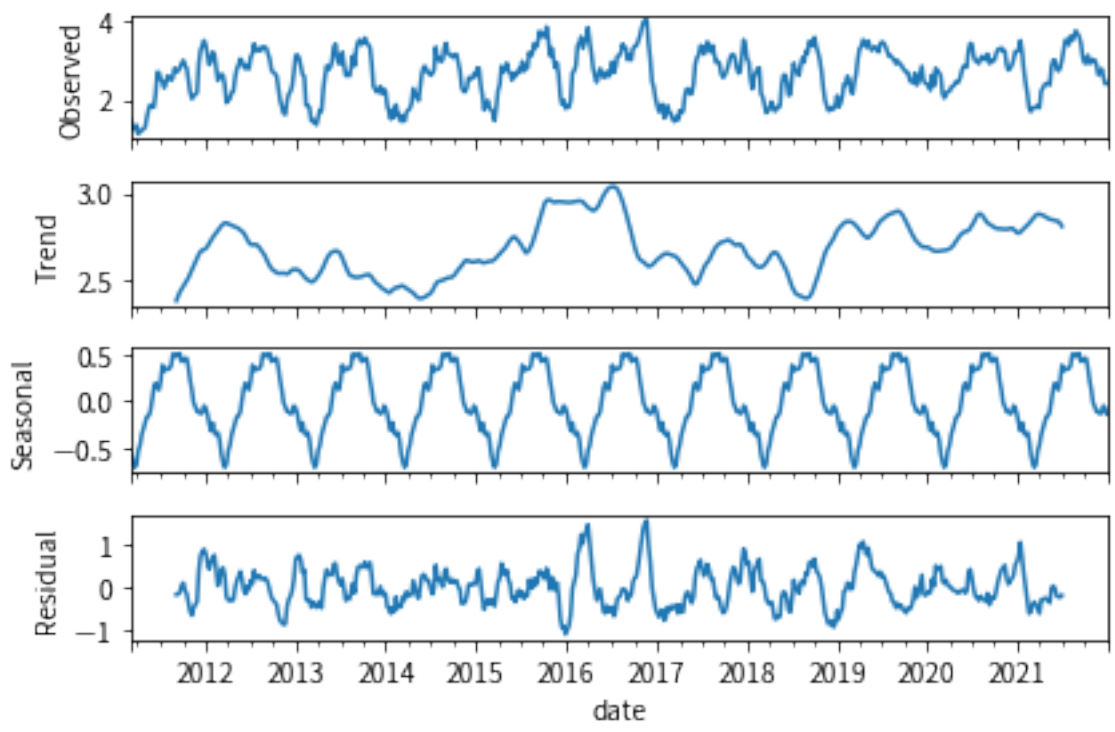


[19]:
```
df['year'] = df.index.year
sns.pairplot(df, hue='year')
plt.show()
```

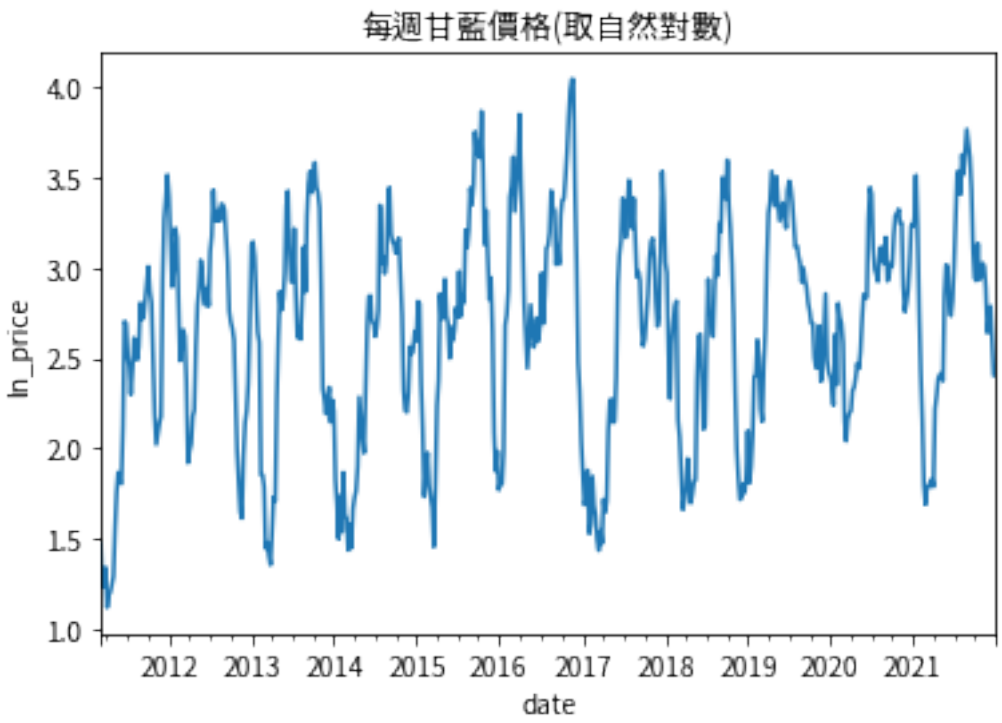


5.2 Decomposition

[20]:
```
decomp_results = seasonal_decompose(df.ln_price)
decomp_results.plot()
plt.show()
```



[21]:
```
df.ln_price.plot()
plt.title('每週甘藍價格 (取自然對數)')
plt.ylabel('ln_price')
plt.show()
```



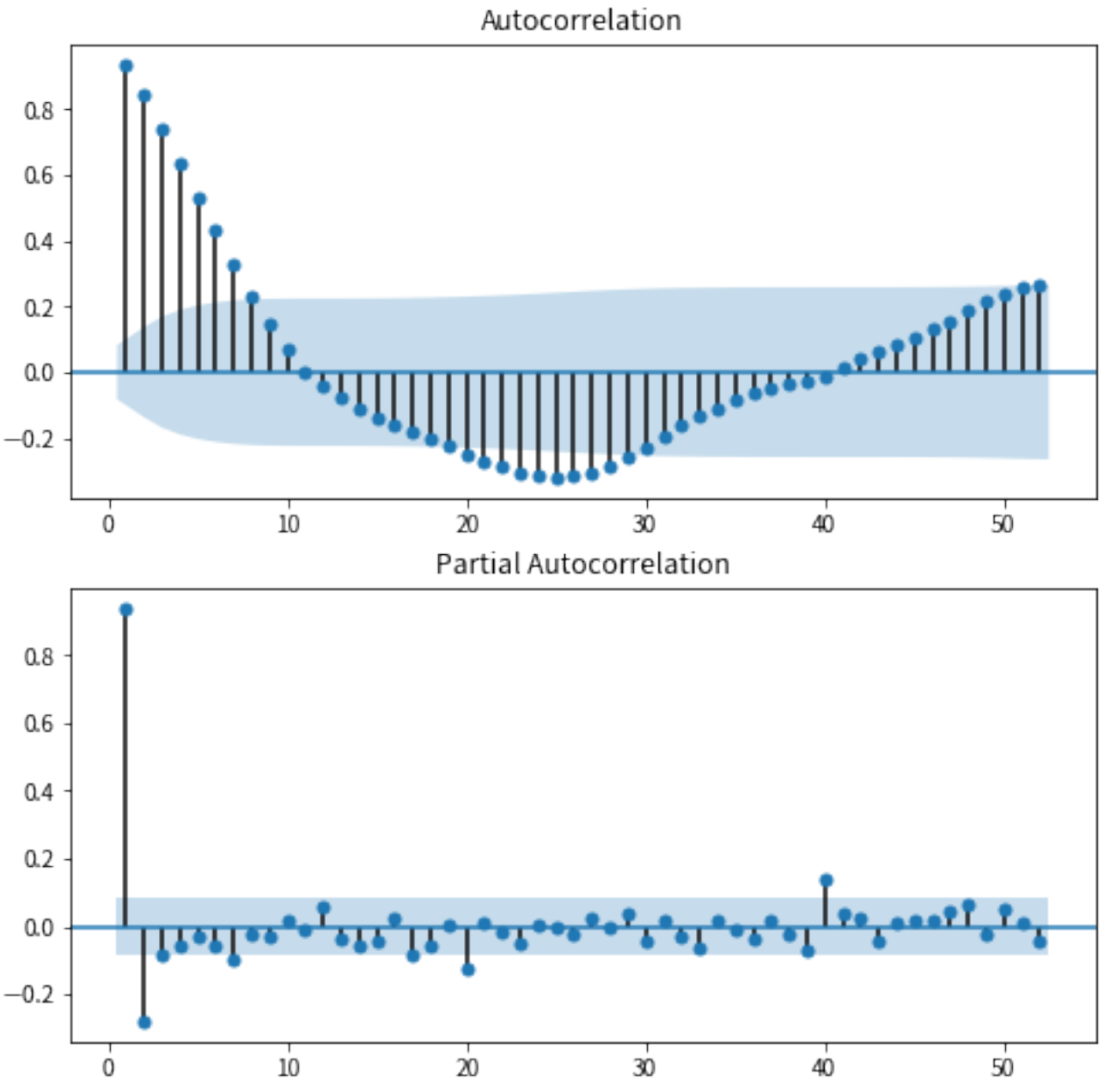
5.3 ADF TEST

[22]:
```
# print(adfuller(df.price))
print(adfuller(df.ln_price))
```

```
(-6.86993697048007, 1.524778106779821e-09, 6, 558, {'1%': -3.4421235439968862,
'5%': -2.866733577794069, '10%': -2.569536010842615}, -218.19823520770672)
```

5.4 ACF 與 PACF

[23]:
```
# Create figure
fig, (ax1, ax2) = plt.subplots(2,1, figsize=(8,8))

# Make ACF plot
plot_acf(df.ln_price, lags=52, zero=False, ax=ax1)

# Make PACF plot
plot_pacf(df.ln_price, lags=52, zero=False, ax=ax2, method='ywm')
plt.show()
```



6 模型

6.1 SARIMAX

$SARIMAX(p,d,q) \times (P,D,Q) : Seasonal + ARIMA + Exogenous$
$AR(p) : Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots + \beta_p Y_{t-p} + u_t$
$MR(q) : Y_t = \theta_0 + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} + u_t$
$ARIMR(p,d,q) : (1 - \sum_{i=1}^{p} \beta_i L^i)(1-L)^d Y_t = (1 - \sum_{i=1}^{q} \theta_i L^i) u_t$, where L stands for Lag operator.

[24]:
```
%%capture
results = []
#for tr in trs:
    #result = pm.auto_arima(tr.ln_price, X=tr[['LA_num', 'Temperature', 'Precp']], seasonal=True, D=1, m=52)
    ##WEEK:  SARIMAX(3, 0, 0)x(2, 1, 0, 52)      ##MONTH: SARIMAX(2, 0, 0)x(2, 1, 0, 12)
for tr in trs:
    model = SARIMAX(tr.ln_price, exog=tr[['LA_num', 'Temperature', 'Precp']],
                    order=(3, 0, 0), seasonal_order=(2, 1, 0, 52))
    result = model.fit()
```

```
    results.append(result)
```

[25]:
```
print(results[-1].summary())
```

```
                                Statespace Model Results
==========================================================================================
Dep. Variable:                            ln_price   No. Observations:                  556
Model:             SARIMAX(3, 0, 0)x(2, 1, 0, 52)   Log Likelihood                -15.036
Date:                            Fri,  4 Mar 2022   AIC                             48.073
Time:                                    01:37:33   BIC                             86.076
Sample:                                03-11-2011   HQIC                            62.980
                                     - 10-29-2021
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
LA_num       -9.42e-06   2.07e-05     -0.456      0.648   -4.99e-05    3.11e-05
Temperature    -0.0342      0.007     -4.897      0.000      -0.048      -0.020
Precp          -0.0003      0.002     -0.186      0.852      -0.003       0.003
ar.L1           1.0221      0.078     13.065      0.000       0.869       1.175
ar.L2          -0.0544      0.120     -0.454      0.649      -0.289       0.180
ar.L3          -0.1000      0.081     -1.238      0.216      -0.258       0.058
ar.S.L52       -0.6503      0.077     -8.436      0.000      -0.801      -0.499
ar.S.L104      -0.4360      0.074     -5.907      0.000      -0.581      -0.291
sigma2          0.0885      0.009      9.850      0.000       0.071       0.106
==========================================================================================
Ljung-Box (Q):                       47.67   Jarque-Bera (JB):                 3.78
Prob(Q):                              0.19   Prob(JB):                         0.15
Heteroskedasticity (H):               0.80   Skew:                             0.21
Prob(H) (two-sided):                  0.16   Kurtosis:                         2.94
==========================================================================================
Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```

6.1.1 訓練資料

[26]:
```
def sarimax_forcast(tr, va=pd.DataFrame(), exo=['LA_num', 'Temperature', 'Precp'], ci=0.68):
    """
    Args:
      tr: Training Set
      va: Validation Set
      exo: Exog. List
      ci: Confidence Interval
    Returns:
      fcst: Forcast Dataframe
    """
    # model = SARIMAX(tr.ln_price, exog=tr[exo],
    #                 order=(3,0,0), seasonal_order=(2, 1, 0, 52))
    # result = model.fit()
    result = results[-1]

    preiods = len(va)
    if preiods == 0:
        predicted = result.get_prediction()
    else:
        predicted = result.get_forecast(steps=preiods, exog=va[exo])

    mean = pd.DataFrame({'predicted_mean':predicted.predicted_mean})
    conf = predicted.conf_int(alpha=1-ci)  # 68%
```

```
    fcst = pd.concat([mean, conf], axis=1)
    return fcst
```

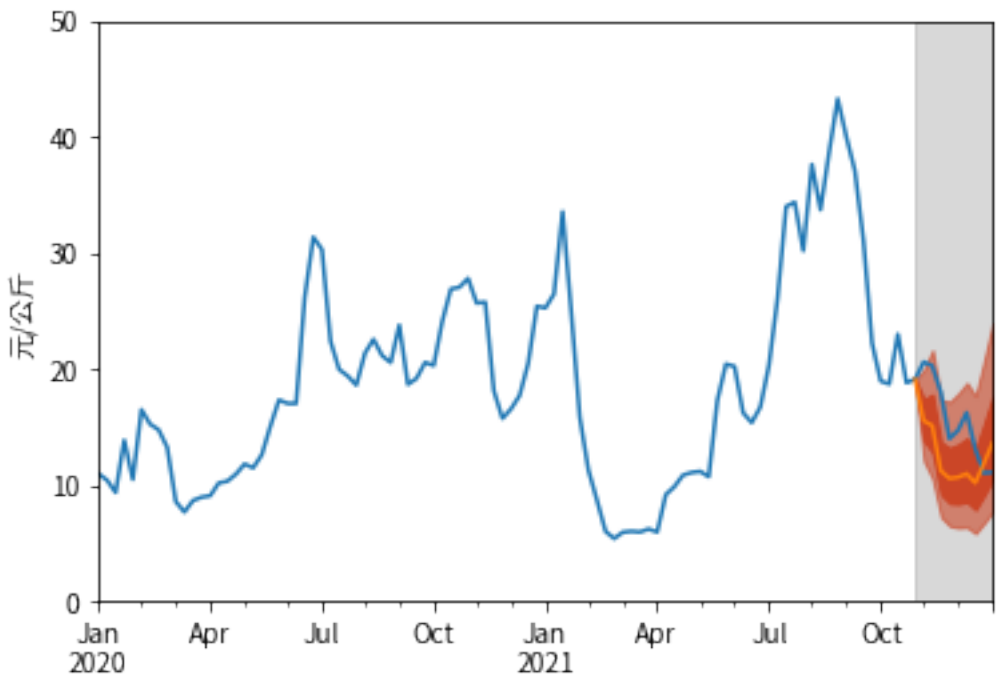
[27]:
```
def connectpoint(tr):
    """
    Args:
      tr: Training Set
    Returns:
      lastp: Price of last date
      lastci: Price lower & upper Bound of last date
    """
    lastp = tr.price[-1:]
    lastci = tr[['price', 'price']][-1:]
    lastci.columns = ['lower price', 'upper price']
    lastci.index.name = 'ds'
    return lastp, lastci
```

[28]:
```
def fanchart(tr, va, fcst30, fcst60, start='2020'):
    """
    Args:
      tr: Training Set
      va: Validation Set
      fcst30: DataFrame of 30% CI
      fcst60: DataFrame of 60% CI
      start: Start Time
    Returns:
      plt.show()
    """
    lastp, lastci = connectpoint(tr)

    yhat = fcst30.iloc[:, 0]

    point_est = np.exp(yhat)
    point_est = point_est.append(lastp).sort_index()

    fig, ax = plt.subplots()
    tr[start:].price.plot(ax=ax, legend=False)
    va.price.append(lastp).sort_index().plot(color='C0')

    point_est.plot()
    plt.axvspan(point_est.index.min(), point_est.index.max(), color='grey', alpha=0.3)

    conf30 = fcst30.iloc[:, 1:]
    conf60 = fcst60.iloc[:, 1:]
    confs = [conf30, conf60]

    for i, conf in enumerate(confs):
        conf = np.exp(conf)
        conf.columns = ['lower price', 'upper price']
        conf = conf.append(lastci).sort_index()
        plt.fill_between(conf.index, conf['lower price'], conf['upper price'],
                         color='xkcd:tomato red', alpha=(1-i/2.5), facecolor='black')

    ax.set_xlabel('')
    ax.set_ylabel('元/公斤')
    ax.set_ylim(0, 50)
    return plt.show()
```

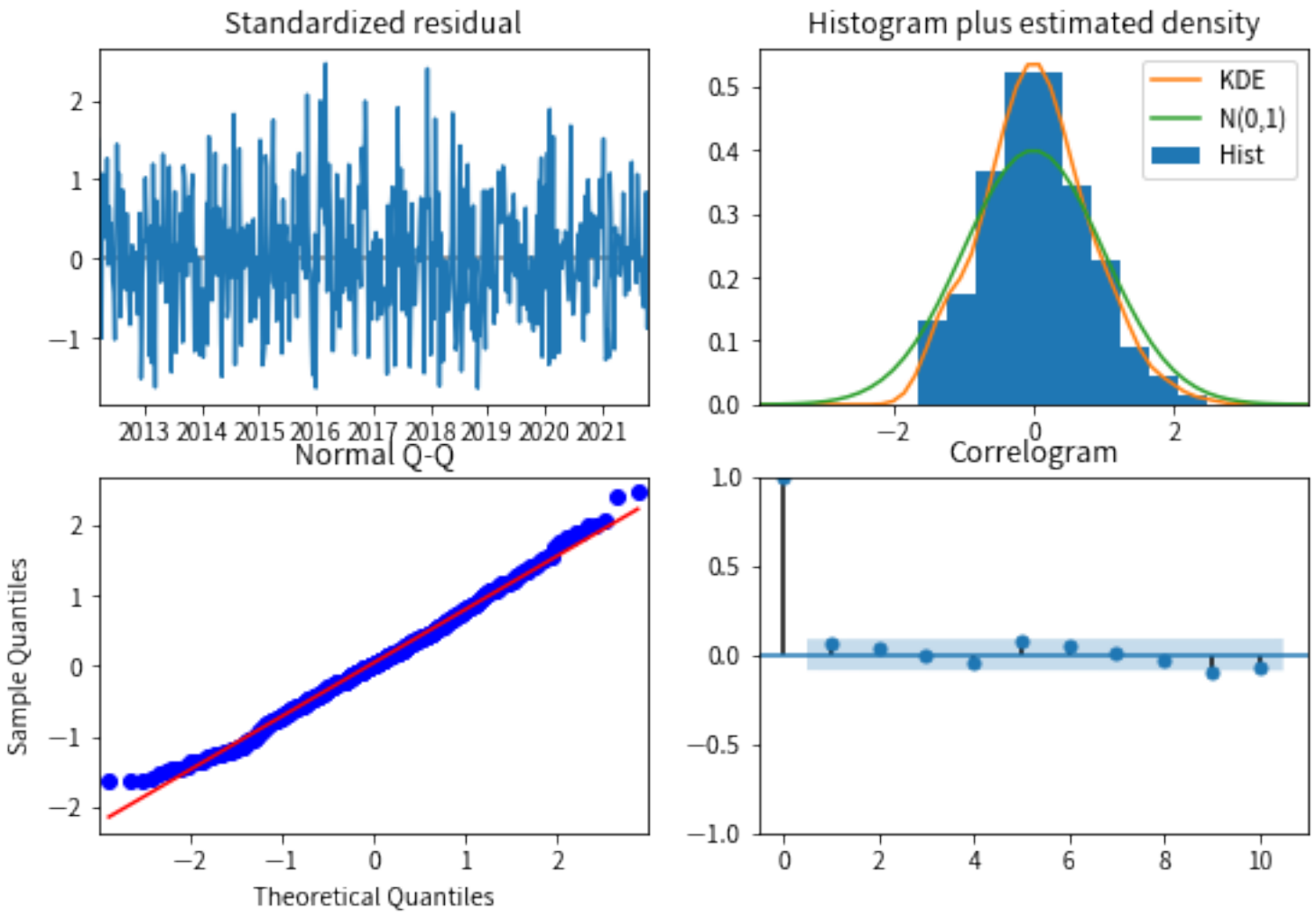
[ ]:
```
# sarimax_forcast(trs[-1])
```

6.1.2 驗證資料

[29]:
```
fcst30 = sarimax_forcast(tr=trs[-1], va=vas[-1], ci=0.3)
fcst60 = sarimax_forcast(tr=trs[-1], va=vas[-1], ci=0.6)

fanchart(trs[-1], vas[-1], fcst30, fcst60)
```



[30]:
```
results[-1].plot_diagnostics(figsize=(9,6))
plt.show()
```



6.2 fbporphet

$y(t) = g(t) + s(t) + h(t) + e(t)$

$g(t)$: trend models non-periodic changes.

$s(t)$: seasonality presents periodic changes.

$h(t)$: effects of holidays with irregular schedules.

$e(t)$: covers idiosyncratic changes not accommodated by the model.

6.2.1 設定資料

[31]:
```
def setdata(df):
    """
    Args:
      df: Source DataFrame
    Returns:
      df: Result DataFrame
    """
    df = df.reset_index()
    df.columns = ['ds', 'price', 'LA_num', 'Temperature', 'Precp', 'ln_price']
    df['y'] = df.ln_price
    return df
```

[32]:
```
trps, vaps = [], []
for tr, va in zip(trs, vas):
    trps.append(setdata(tr))
    vaps.append(setdata(va))
```

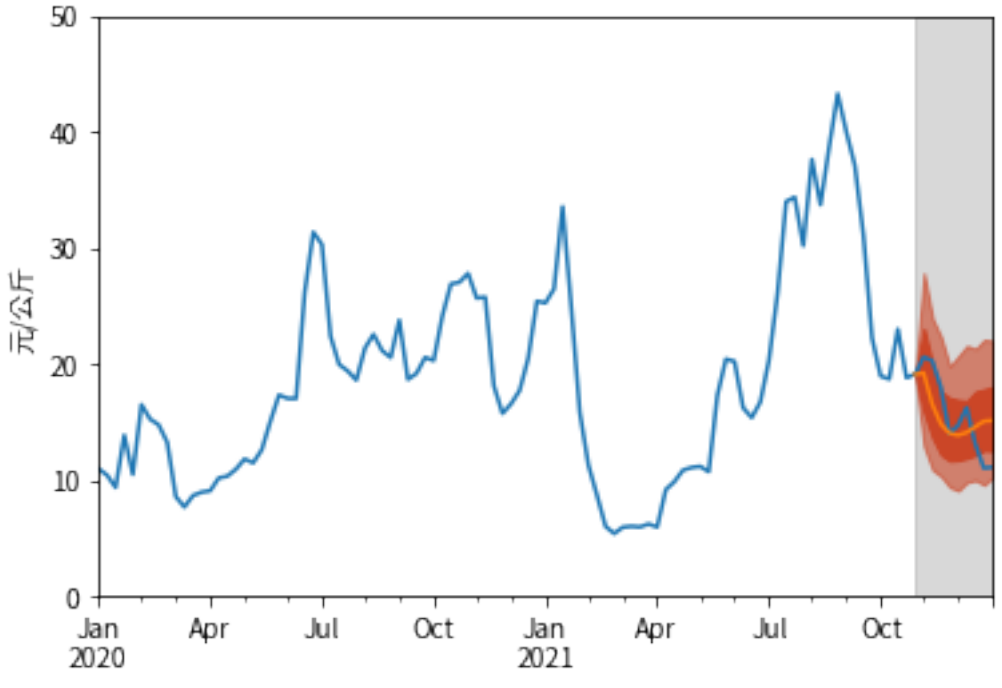
6.2.2 訓練資料

[33]:
```
def prophet_forecast(tr, va=pd.DataFrame(), exo=['LA_num', 'Temperature', 'Precp'], ci=0.68):
    """
    Args:
      tr: Training Set
      va: Validation Set
      exo: Exog. List
      ci: Confidence Interval
    Returns:
      fcst: Forecast Dataframe
    """
    m = Prophet(seasonality_mode='multiplicative', interval_width=ci,
                yearly_seasonality=True, weekly_seasonality=False, daily_seasonality=False)
    for x in exo:
        m.add_regressor(x)
    cols = ['ds', 'y'] + exo
    m.fit(tr[cols])

    preiods = len(va)
    if preiods == 0:
        predict = m.make_future_dataframe(periods=preiods, freq='W-Fri', include_history=True)
        for x in exo:
            predict[x] = tr[[x]].reset_index(drop=True)
    else:
        predict = m.make_future_dataframe(periods=preiods, freq='W-Fri', include_history=False)
        for x in exo:
            predict[x] = va[[x]].reset_index(drop=True)
    fcst = m.predict(predict)
    fcst = fcst.set_index(fcst.ds, drop=True)
    return fcst[['yhat', 'yhat_lower', 'yhat_upper']]
```

[ ]:
```
# prophet_forecast(trps[-1])
```

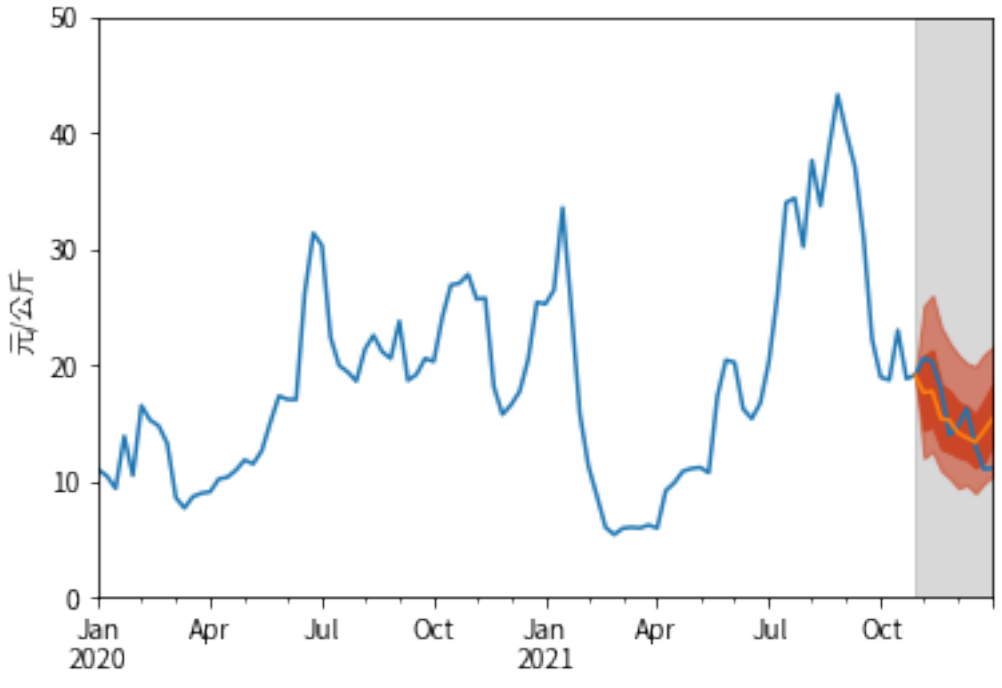
6.2.3 驗證資料 - 無外生變數

[34]:
```
fcst30 = prophet_forecast(tr=trps[-1], va=vaps[-1], exo=[], ci=0.3)
fcst60 = prophet_forecast(tr=trps[-1], va=vaps[-1], exo=[], ci=0.6)

fanchart(trs[-1], vas[-1], fcst30, fcst60)
```



6.2.4 驗證資料

[35]:
```
fcst30 = prophet_forecast(tr=trps[-1], va=vaps[-1], ci=0.3)
fcst60 = prophet_forecast(tr=trps[-1], va=vaps[-1], ci=0.6)

fanchart(trs[-1], vas[-1], fcst30, fcst60)
```



7 評估

7.1 準備評估資料

7.1.1 訓練資料

[36]:
```
dfins = pd.DataFrame({'y_real': trs[-1].price,
                      'y_sarimax': np.exp(results[-1].get_prediction().predicted_mean),
                      'y_prophet': np.exp(prophet_forecast(trps[-1]).yhat)})
dfins.tail()
```

[36]:

	y_real	y_sarimax	y_prophet
2021-10-01	18.968285	22.790067	33.187853
2021-10-08	18.718567	16.033195	24.749402
2021-10-15	23.008165	17.930942	21.109855
2021-10-22	18.830983	24.478487	21.356169
2021-10-29	19.119969	16.749888	19.594124

7.1.2 驗證資料

[37]:
```
dfoos = pd.DataFrame({'y_real': vas[-1].price,
                      'y_sarimax': np.exp(sarimax_forcast(tr=trs[-1], va=vas[-1], ci=0.3).predicted_mean),
                      'y_prophet': np.exp(prophet_forecast(tr=trps[-1], va=vaps[-1]).yhat)})
dfoos.head()
```

[37]:

	y_real	y_sarimax	y_prophet
2021-11-05	20.615359	15.554690	17.644919
2021-11-12	20.288235	15.138622	17.752219
2021-11-19	18.010898	11.244562	15.407024
2021-11-26	14.008714	10.591525	15.251011
2021-12-03	14.733079	10.686173	14.177256

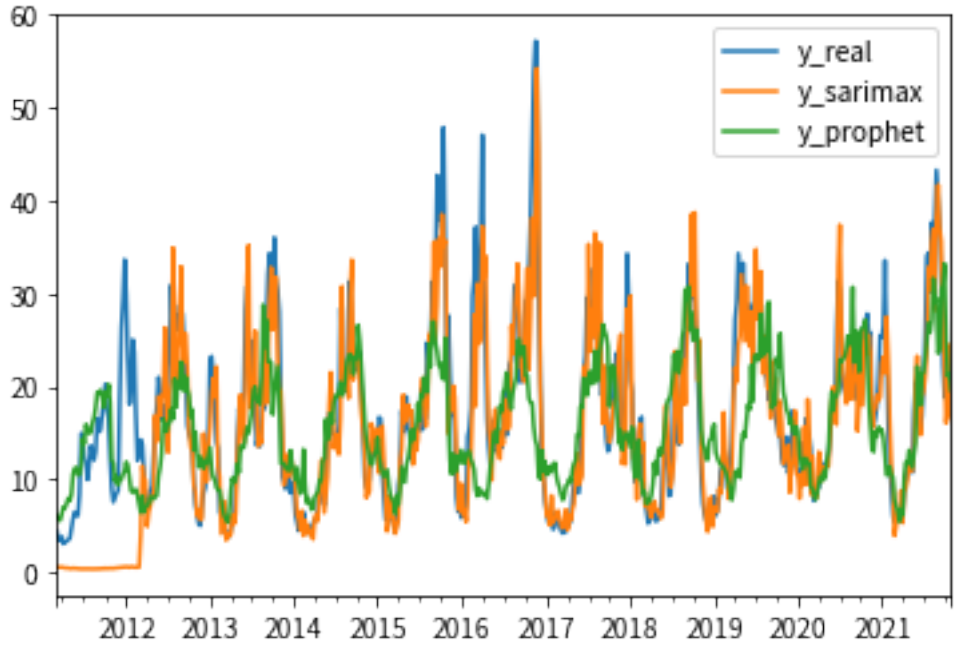
7.2 評估標準

[38]:
```
from sklearn.metrics import mean_squared_error, mean_absolute_error, mean_absolute_percentage_error
```

- $\text{MSE}(Y, \widehat{Y}) = \frac{1}{T}\sum_{t=1}^{T}(Y_t - \widehat{Y}_t)^2$
- $\text{MAE}(Y, \widehat{Y}) = \frac{1}{T}\sum_{t=1}^{T}|Y_t - \widehat{Y}_t|$
- $\text{MAPE}(Y, \widehat{Y}) = \frac{1}{T}\sum_{t=1}^{T}\frac{|Y_t - \widehat{Y}_t|}{max(u_t, |Y_t|)}$

7.3 訓練資料

[39]:
```
dfins.plot()
plt.show()
```



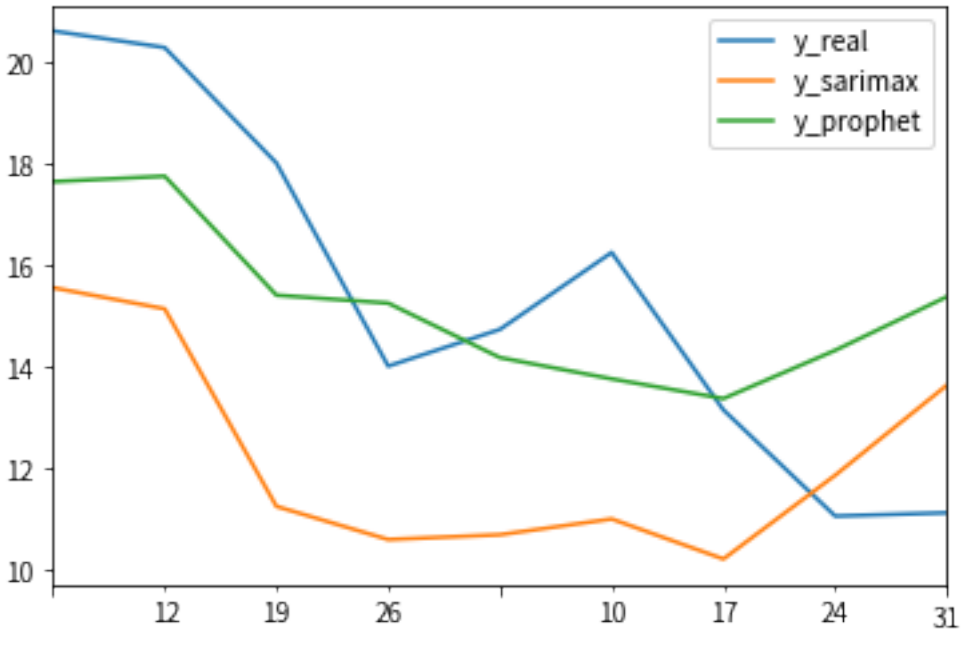
[40]:
```
mse = [mean_squared_error(dfins.y_real, dfins.y_sarimax),
       mean_squared_error(dfins.y_real, dfins.y_prophet)]
mae = [mean_absolute_error(dfins.y_real, dfins.y_sarimax),
       mean_absolute_error(dfins.y_real, dfins.y_prophet)]
mape = [mean_absolute_percentage_error(dfins.y_real, dfins.y_sarimax),
        mean_absolute_percentage_error(dfins.y_real, dfins.y_prophet)]
dfv_ins = pd.DataFrame({'MSE': mse,
                        'MAE': mae,
                        'MAPE': mape}, index=['SARIMAX', 'prophet'])
dfv_ins
```

[40]:

	MSE	MAE	MAPE
SARIMAX	38.315079	4.026199	0.256260
prophet	63.482737	5.554545	0.366049

7.4 驗證資料

[41]:
```
dfoos.plot()
plt.show()
```



[42]:
```
mse = [mean_squared_error(dfoos.y_real, dfoos.y_sarimax),
       mean_squared_error(dfoos.y_real, dfoos.y_prophet)]
rmse = [np.sqrt(mean_squared_error(dfoos.y_real, dfoos.y_sarimax)),
        np.sqrt(mean_squared_error(dfoos.y_real, dfoos.y_prophet))]
mae = [mean_absolute_error(dfoos.y_real, dfoos.y_sarimax),
       mean_absolute_error(dfoos.y_real, dfoos.y_prophet)]
mape = [mean_absolute_percentage_error(dfoos.y_real, dfoos.y_sarimax),
      mean_absolute_percentage_error(dfoos.y_real, dfoos.y_prophet)]
dfv_oos = pd.DataFrame({'MSE': mse,
                        'RMSE': rmse,
                        'MAE': mae,
                        'MAPE': mape}, index=['SARIMAX', 'prophet'])
dfv_oos
```

[42]:

	MSE	RMSE	MAE	MAPE
SARIMAX	18.794983	4.335318	3.994496	0.248816
prophet	6.550445	2.559384	2.238885	0.154340

[43]:
```
# log(Price)
mse = [mean_squared_error(np.log(dfoos.y_real), np.log(dfoos.y_sarimax)),
       mean_squared_error(np.log(dfoos.y_real), np.log(dfoos.y_prophet))]
rmse = [np.sqrt(mean_squared_error(np.log(dfoos.y_real), np.log(dfoos.y_sarimax))),
        np.sqrt(mean_squared_error(np.log(dfoos.y_real), np.log(dfoos.y_prophet)))]
mae = [mean_absolute_error(np.log(dfoos.y_real), np.log(dfoos.y_sarimax)),
       mean_absolute_error(np.log(dfoos.y_real), np.log(dfoos.y_prophet))]
mape = [mean_absolute_percentage_error(np.log(dfoos.y_real), np.log(dfoos.y_sarimax)),
      mean_absolute_percentage_error(np.log(dfoos.y_real), np.log(dfoos.y_prophet))]
dfv_oos = pd.DataFrame({'MSE': mse,
                        'RMSE': rmse,
                        'MAE': mae,
                        'MAPE': mape}, index=['SARIMAX', 'prophet'])
dfv_oos
```

[43]:

	MSE	RMSE	MAE	MAPE
SARIMAX	0.092360	0.303908	0.284876	0.103418
prophet	0.030591	0.174902	0.148365	0.056106

Lewis(1982)
MAPE < 10%：Highly accurate forcasting
10%< MAPE < 20%：Good forecasting
20%< MAPE < 50%：Reasonable forecasting
MAPE > 50%：Week and inaccurate forecasting

Those Lewis numbers are fairly arbitrary, you can't just say that a 20% error is good forecasting because some guy wrote it in a book 40 years ago.The acceptable margin or error completely depends on the problem domain. In some situations a model that gives a 20% error will be great, in others it will be unusable. I know its tempting to rely on general rules like the ones you posted because they feel 'objective', but they are ultimately arbitrary and can't override common sense and domain expertise.

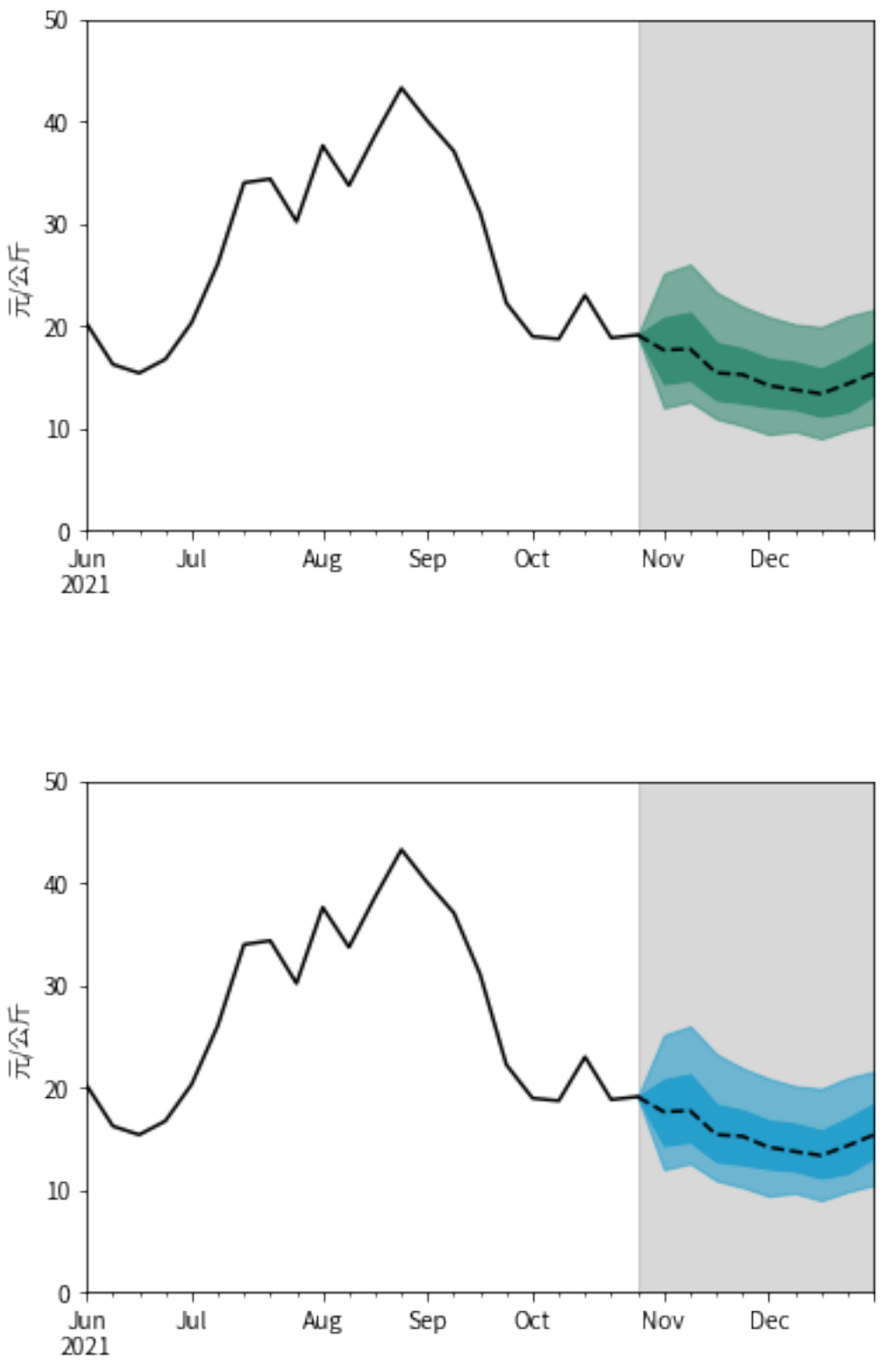
8 扇形圖

8.1 定義 fanchart function

[44]:
```
def fanchart(tr, va, fcst30, fcst60, start='2020', color='#199370'):
    """
    Args:
      tr: Training Set
      va: Validation Set
      yhat: Forecasting Price
      lastp: Price of last date
      lastci: Confidence Interval of last date
    Returns:
      plt.show()
    """
    lastp, lastci = connectpoint(tr)

    yhat = fcst30.iloc[:, 0]

    point_est = np.exp(yhat)
    point_est = point_est.append(lastp).sort_index()

    fig, ax = plt.subplots()
    tr[start:].price.plot(ax=ax, legend=False, color='black')
    # va.price.append(lastp).sort_index().plot(color='C0')

    point_est.plot(color='black', linestyle='--')
    plt.axvspan(point_est.index.min(), point_est.index.max(), color='grey', alpha=0.3)

    conf30 = fcst30.iloc[:, 1:]
    conf60 = fcst60.iloc[:, 1:]
    confs = [conf30, conf60]

    for i, conf in enumerate(confs):
        conf = np.exp(conf)
        conf.columns = ['lower price', 'upper price']
        conf = conf.append(lastci).sort_index()
        plt.fill_between(conf.index, conf['lower price'], conf['upper price'],
                         color=color, alpha=(1-i/2.5), facecolor='black')

    ax.set_xlabel('')
    ax.set_ylabel('元/公斤')
    ax.set_ylim(0, 50)
    return plt.show()
```

8.2 繪製

[45]:
```
fanchart(trs[-1], vas[-1], fcst30, fcst60, start='2021-06')
fanchart(trs[-1], vas[-1], fcst30, fcst60, start='2021-06', color='#00adee')
```



9 TODO

9.1 Six Validation

[ ]:

9.2 Test Set

[ ]: